Last Name _____

First Name _____

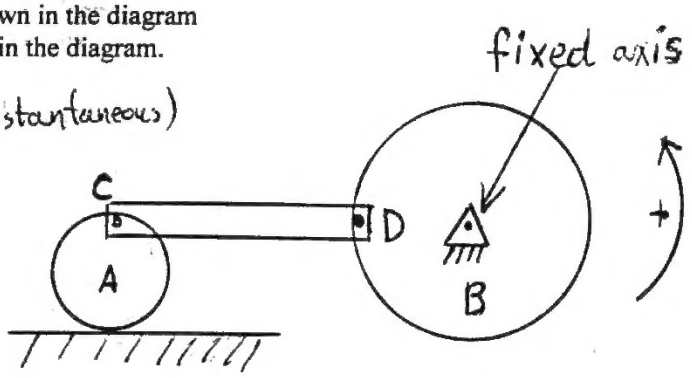
Student number _____

MIE 200F - Quiz number 6 - October 29/98
quiz duration = 15 minutes

The wheel "A" of radius 2 meters is linked by a bar of length 9 meters to a wheel "B" of radius 4 meters. Wheel "B" can spin about its fixed axis. Wheel "A" can roll on the ground, but there is no slippage between wheel "A" and the ground. At the instant shown in the diagram, the bar is horizontal and wheel "B" has a constant angular velocity of $\omega_B = 6\ s^{-1}$ in the positive direction.

(a) find the angular velocity of wheel "A" at the instant shown in the diagram

(b) find the angular velocity of the bar at the instant shown in the diagram.



(a)(i) Find center of zero velocity (instantaneous) for Bar CD, on dotted lines

Lines cross at point C ⇒ "C" has zero velocity ⇒ $\omega_A = 0$

or // (ii) $\vec{V}_C = \vec{V}_D + \vec{V}_{C/D}$

$\rightarrow = \downarrow + \uparrow$

Horizontal vector = sum of 2 vertical vectors

⇒ $\vec{V}_C = 0$ ⇒ $\omega_A = 0$

4

(b) $|\vec{V}_D| = \omega_B r_{BD} = (6)(4) = 24\ s^{-1}$

$\omega_{CD} = \dfrac{|\vec{V}_C - \vec{V}_D|}{r_{CD}} = \dfrac{24}{9} = 2.67\ s^{-1}$ ↓

or $\omega_{CD} = -2.67\ s^{-1}$

3

3

10

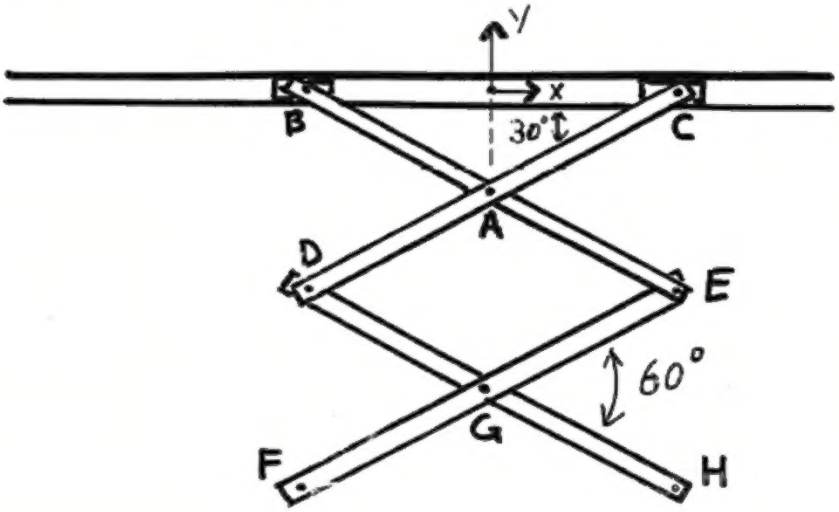
The system of bars causes points "A" and "G" to move in a purely vertical direction. Each bar has a length of 10 meters, with points "A" and "G" at the bar midpoints. The blocks at points "B" and "C" can slide in the horizontal direction only. At the instant shown, the velocities of "B" and "C" are constant, and equal to: $\vec{v}_B = -2$ m/s $\hat{i}$, and $\vec{v}_C = 2$ m/s $\hat{i}$.

(a) Determine the x-y coordinates of the instantaneous center of zero velocity for bar BE, at the instant shown.

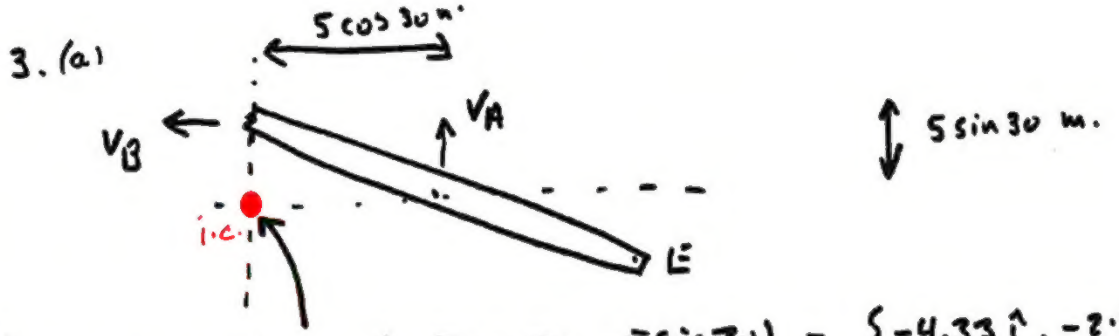
(b) Determine ω for the bar CD at the instant shown.

(c) Find the ratio of $|\vec{a}_G|$ to $|\vec{a}_A|$.

(d) Find the acceleration of "A" in x-y coordinates.



3. (a)

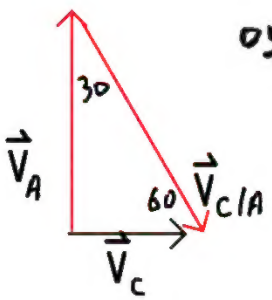


instaneous center = $(-5\cos 30, -5\sin 30) = \{-4.33\,\hat{i}, -2.5\,\hat{j}\}$ m

(b) ω: using center of zero velocity,

$\omega = V_C / r_{OC}$, where "O" is center of zero velocity for CD

$= \frac{2}{2.5} = 0.8\ s^{-1}$ ↻



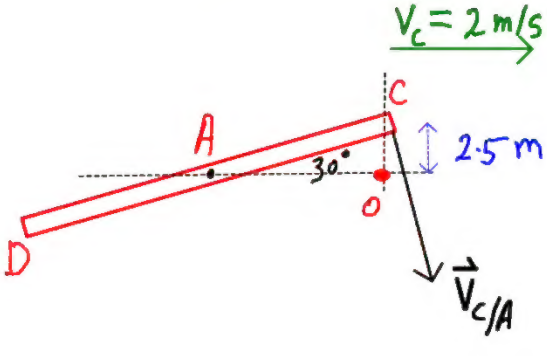
or

$|\vec{V}_{C/A}| = |\vec{V}_C| / \cos 60 = 4$ m/s.

2

but $|\vec{V}_{C/A}| = |\omega_{AC}\, r_{AC}|$ → $\omega_{AC} = 0.8$

4



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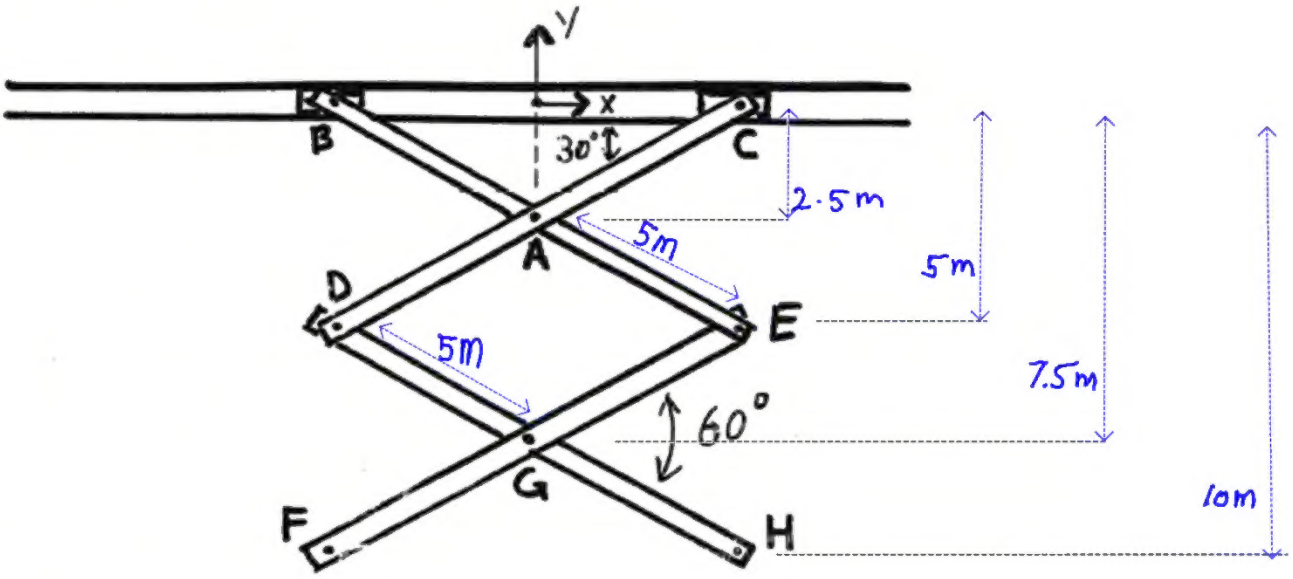
(d) Find the acceleration of "A" in x-y coordinates.



(c) From diagram, the vertical separations between C, A, E, G, and H are all equal.
i.e, the vertical position of "A" is always halfway between C and E

the vertical position of E is always halfway between C and H, etc.

To maintain this equal separation, their vertical speeds must vary linearly,

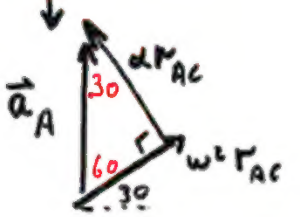
$V_C : V_A : V_E : V_G : V_H = 0:1:2:3:4$ (vertical components)

similar relationship holds for acceleration. (vertical components)

$\Rightarrow |\vec{a}_G| : |\vec{a}_A| = 3:1$

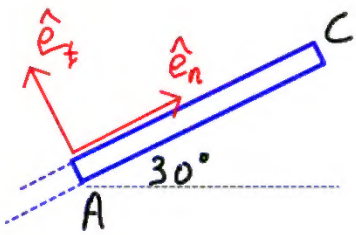
(d) $\vec{a}_A = \vec{a}_C + (a_{A/C})_n \hat{e}_n + (a_{A/C})_t \hat{e}_t$

$\vec{a}_A = \vec{a}_C^{\,0} + \omega^2 r_{AC} + \alpha r_{AC}$



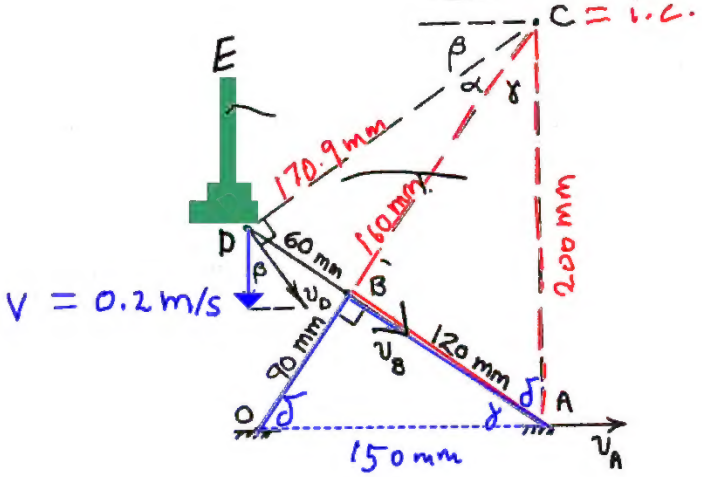
$\omega^2 r_{CA} = (.8)^2(5) = 3.2$ m/s²

$a_A = \frac{3.2 \text{ m/s}^2}{\cos 60} = 6.4 \text{ m/s}^2 \, \hat{j}$

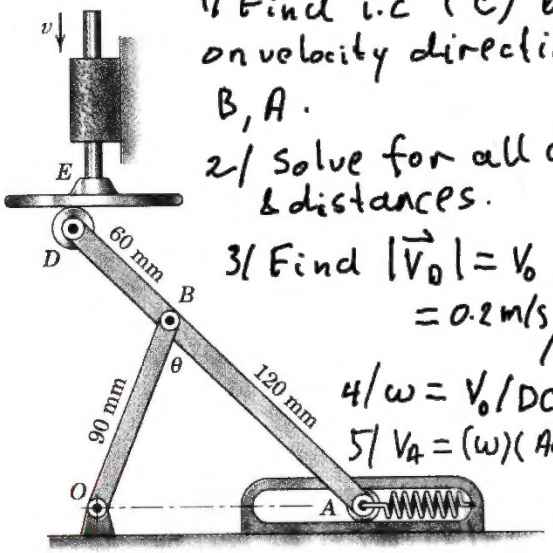


week 8(e)

Motion of the roller A against its restraining spring is controlled by the downward motion of the plunger E. For an interval of motion the velocity of E is v = 0.2 m/s. Determine the velocity of A when θ becomes 90°.



1/ Find i.c ('c') based on velocity directions of B, A.
2/ Solve for all angles & distances.
3/ Find $|\vec{V}_D| = V_o = 0.2 \text{ m/s} / \cos\beta$
4/ $\omega = V_o / DC$
5/ $V_A = (\omega)(AC)$



$\theta = 90°$

Consider the bar ABD when θ equals ninety degrees. We know the following:
(a) We know the vertical component of the velocity of D, which we give the symbol v.
(b) We know the direction of the velocity of B (it must be perpendicular to OB).
(c) We know the direction of the velocity of A.

Since we know the directions of velocity for two different points (B & D), it makes sense to use the method of instantaneous center of zero velocity. From the attached sketch and trigonometry, we note the following:

trigonometry

A) From Pythagoras, line segment OA is 150 mm. Also, triangle ABC is ~~congruent~~ similar to triangle OBA. Therefore, line segment AC is 150/90 x 120 mm = 200 mm.
Also, line segment BC is 120/90 x 120 mm = 160 mm

B) From Pythagoras, we find that line segment DC = sqrt($60^2 + 160^2$) = 170.9 mm

We need to find ω for ABD; then we can find the velocity of point "A". We can find ω if we know the total velocity V_o of "D". Unfortunately we were only given the vertical component of its velocity, v. We can find V_o if we can get the angle β. Doing some more trigonometry,

C) angle $\gamma = \sin^{-1}(120/200) = 36.9$ degrees; angle $\alpha = \tan^{-1}(60/160) = 20.6$ degrees]

D) $V_o = v/(\cos\beta) = (0.2 \text{ m/s}) / (\cos 32.6 \text{ degrees}) = 0.237 \text{ m/s}$

$\beta = 90 - \gamma - \alpha = 32.6°$

Now we know where the instantaneous center of zero velocity is located, plus the velocity of one point on the bar. That is all that is need to get ω for ABD:

$\delta = 90 - \gamma = 53.1°$

$\omega = V_o / DC = 0.237 / 0.1709 = 1.387 \text{ s}^{-1}$

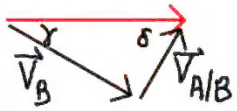
We can now get the velocity of any other point. Specifically,

$V_A = \omega \times AC = 1.387 \text{ s}^{-1} \times 200 \text{ mm} = 277 \text{ mm/s}$

(From the diagram, the direction of "A" must be to the right.)

or:

1/ $\vec{V}_A = \vec{V}_B + \vec{V}_{A/B}$



$\rightarrow \omega = \frac{V_A}{\sin\gamma}\frac{1}{r_{AB}} = C_1 V_A$

2. $\vec{V}_D = \vec{V}_A + \vec{V}_{D/A}$; $\omega r_{DA} = C_1 V_A r_{DA}$

→ Direction of $\vec{V}_D$

→ Magnitude of $\vec{V}_D$